

**Question:**

Calculus: Early Transcendentals 5th Edition: Page 112 Question 33

Calculus: Concepts and Contexts 3rd edition: Page 115 Question 25

Calculus: Concepts and Contexts 2nd edition: Page 118 Question 23

**Solution:**

We know that for any  $x$ ,  $|\cos(20\pi x)| \leq 1$ . It follows that

$$|x^2 \cos(20\pi x)| \leq x^2$$

or equivalently that

$$-x^2 \leq x^2 \cos(20\pi x) \leq x^2.$$

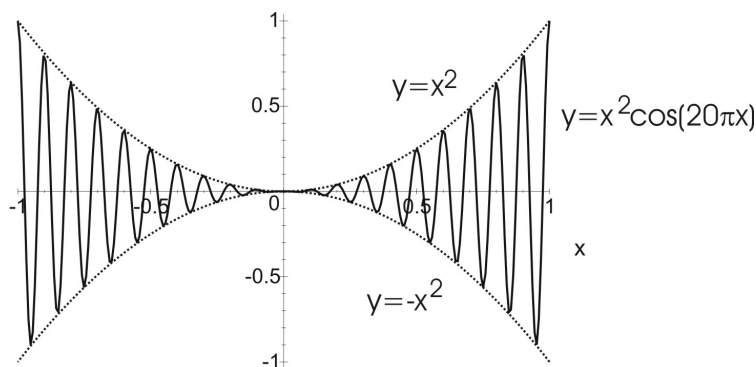
Since

$$\lim_{x \rightarrow 0} -x^2 = 0 = \lim_{x \rightarrow 0} x^2$$

the Squeeze Theorem tells us that

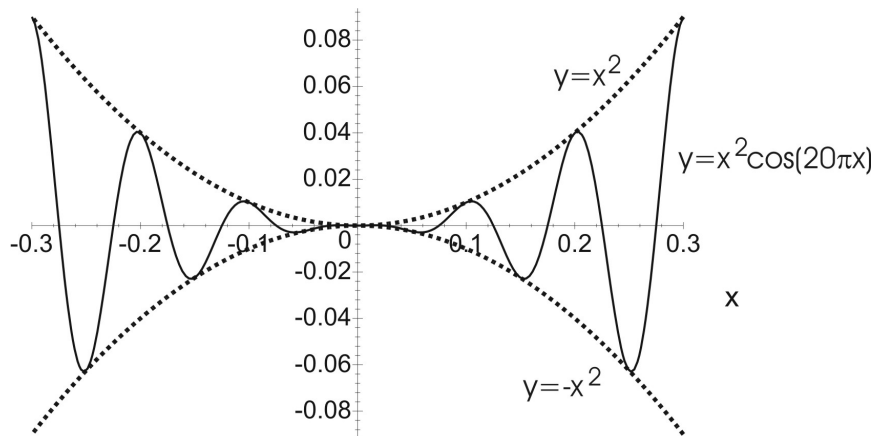
$$\lim_{x \rightarrow 0} x^2 \cos(20\pi x) = 0$$

The graph of  $x^2 \cos(20\pi x)$  on the interval  $[-1, 1]$  is given below. Because of the factor of  $20\pi$ ,  $\cos(20\pi x)$  oscillates many times between 1 and  $-1$  as  $x$  ranges between  $-1$  and 1. Therefore the graph of  $x^2 \cos(20\pi x)$  bounces back and forth between the graphs of  $x^2$  and of  $-x^2$ . To illustrate how the Squeeze Theorem is being used, the graphs of  $x^2$  and  $-x^2$  are also included.



Though your solution did not require it, the graph below is of the same three functions but this time on the interval  $[-.3, .3]$ . This gives additional evidence to support the assertion that

$$\lim_{x \rightarrow 0} x^2 \cos(20\pi x) = 0.$$

**Question:**

Calculus: Early Transcendentals 5th Edition: Page 112 Question 37

Calculus: Concepts and Contexts 3rd edition: Page 115 Question 29

Calculus: Concepts and Contexts 2nd edition: Page 118 Question 27

**Solution:**

We know that for any  $x \neq 0$ ,  $|\cos(\frac{2}{x})| \leq 1$ . It follows that

$$|x^4 \cos(\frac{2}{x})| \leq x^4$$

or equivalently that

$$-x^4 \leq x^4 \cos(\frac{2}{x}) \leq x^4.$$

Since

$$\lim_{x \rightarrow 0} -x^4 = 0 = \lim_{x \rightarrow 0} x^4$$

the Squeeze Theorem tells us that

$$\lim_{x \rightarrow 0} x^4 \cos(\frac{2}{x}) = 0.$$

(Note that the function  $\cos(\frac{2}{x})$  behaves very much like the function  $\sin(\frac{1}{x})$  near  $x = 0$  in that it oscillates between  $-1$  and  $1$  infinitely many times in any interval around  $0$ . The picture below shows the graph of  $\cos(\frac{2}{x})$  together with the graphs of  $\pm x^4$  on the interval  $[-.1, .1]$

