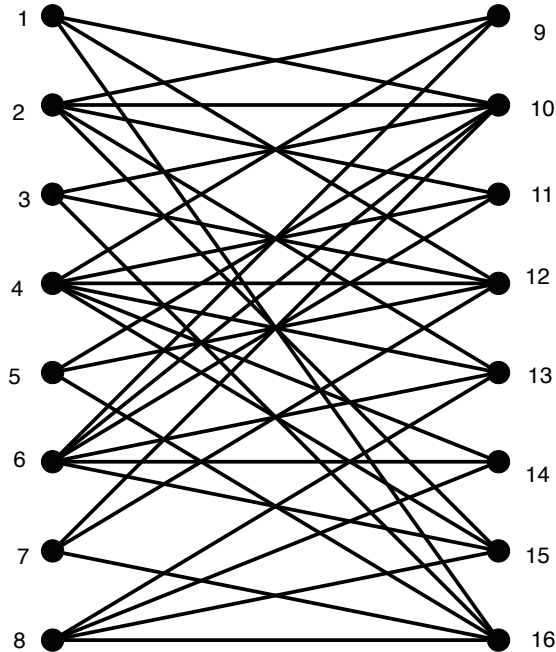


MATH 239 Assignment 9

This assignment is for practice only.

1. Find a maximum matching and a minimum cover in the graph in Figure 1.



2. Find a subset D of $\{1, 2, 3, 4, 5, 6, 7, 8\}$ in the graph from Question 1 such that $|N(D)| < |D|$.
3. Let A be an $n \times n$ matrix. Formulate the problem of finding a largest set of non-zero entries, no two in the same row or same column as a matching problem in some bipartite graph. Interpret in terms of the matrix what a cover in the graph is.
4. Let k be a positive integer and suppose G is a bipartite graph in which every vertex has degree precisely k . Show: (a) that any bipartition (A, B) of G has $|A| = |B|$; (b) G has a perfect matching; and (c) G has k perfect matchings, any two having no edges in common.
5. For each positive integer n , find an example of a bipartite graph with n vertices on both sides of the bipartition, with minimum degree at least 3, and with no matching of size larger than $n/4$.