Problem 1: Consider the identity

$$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$$

a. Give an algebraic proof for the identity.

b. Give a combinatorial proof for the identity.

Problem 2: Give a combinatorial proof of the identity

$$\binom{n+m}{k} = \sum_{i=0}^{k} \binom{m}{i}\binom{n}{k-i}$$

Problem 3: Let $S = \{1, 2, 3, \ldots\}$ and $w(i) = i$ if i is divisible by 3; $w(i) = i-1$ if $i = 1 \bmod 3$ and $w(i) = i-2$ if $i = 2 \bmod 3$. Write the generating function of S with respect to this weight.

Problem 4: Let S be the set of all subset of $\{1, 2, 4, 6, 7\}$ of size two . Define $w(A) = (a-b)^2 = (b-a)^2$ where $A = \{a, b\} \in S$. Write the generating function of S with respect to this weight.