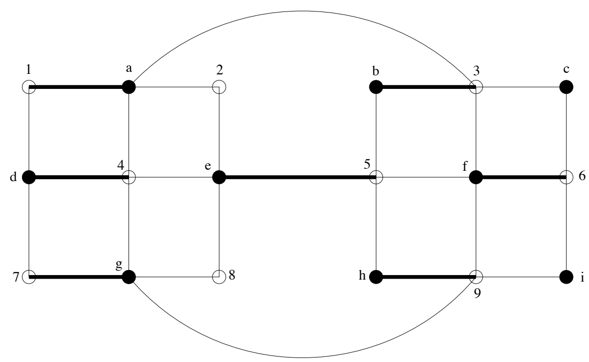
Problem 1: Let M be a maximum matching in a graph G. Let N be any other matching of G. Let H be the subgraph of G whose edges are $(M \cup N) \backslash (M \cap N)$ and whose vertices are the set of vertices of G. Show that if every component of H is a cycle, then N must also be a maximum matching of G.

Problem 2: Let $k \geq 2$. Recall that cycle of length k is a cycle which has k vertices and k edges. Determine the number of perfect matchings and the number of maximum matchings in a cycle of length k.

Problem 3: Apply the bipartite matching algorithm to find a maximum matching and a minimum cover of the following graph:



Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, and let $B = \{a, b, c, d, e, f, g, h, i\}$. The thick edges are the matching edges.

Problem 4: Let the graph T_n be defined for $n \geq 1$ as follows.

1. T_1 is the cycle on 3 vertices,
2. For $n \geq 2$, T_n consists of two copies of T_{n-1}, with additional edges added so that corresponding vertices in the two copies of T_{n-1} are adjacent. More formally, the vertices of T_n are

$$V(T_n) = \{(v, i) : v \in V(T_{n-1}), i \in \{a, b\}\}$$

with adjacency condition $(v, i) \sim (u, j)$ if and only if either $i = j$ and $u \sim v$ in T_{n-1}, or $u = v$ and $i \neq j$.

Prove that T_n has a Hamilton cycle for $n \geq 1$.